

MEMO GR 12 SEQUENCES AND SERIES/ GR 12 RYE EN REEKSE

May/June 2024/ Mei/Junie 2024

QUESTION 2/VRAAG 2

2.1.1	$r = \frac{1}{2}$ <p>Yes, because $-1 < \frac{1}{2} < 1$</p>	<p>✓ $r = \frac{1}{2}$</p> <p>✓ answer with reason</p> <p>(2)</p>
2.1.2	$S_{\infty} = \frac{a}{1-r}$ $S_{\infty} = \frac{4}{1-\frac{1}{2}}$ $\therefore S_{\infty} = 8$	<p>✓ substitution</p> <p>✓ answer</p> <p>(2)</p>
2.2	$\sum_{p=k}^{10} 3^{p-1} = 3^{k-1} + 3^{k+1-1} + 3^{k+2-1} + \dots + 3^9$ $= 3^{k-1} + 3^k + 3^{k+1} + \dots + 3^9$ $S_n = \frac{a(r^n - 1)}{r - 1}$ $29\,520 = \frac{3^{k-1}(3^{11-k} - 1)}{3 - 1}$ $3^{10} - 3^{k-1} = 59\,040$ $3^{k-1} = 9$ $k - 1 = 2$ $\therefore k = 3$	<p>✓ $a = 3^{k-1}$</p> <p>✓ $r = 3$</p> <p>✓ $n = 11 - k$</p> <p>✓ substitution</p> <p>✓ answer</p> <p>(5)</p>
		[9]

QUESTION 3/VRAAG 3

<p>3.1.1</p>	$\begin{array}{cccc} & & 3 & ; & 7 & ; & 12 & ; & 18 \\ & & \vee & & \vee & & \vee & & \\ \text{First diff:} & & 4 & ; & 5 & ; & 6 & & \\ & & \vee & & \vee & & & & \\ \text{Second diff:} & & 1 & ; & 1 & & & & \end{array}$ $2a=1$ $a = \frac{1}{2}$ $3a+b=4$ $3\left(\frac{1}{2}\right)+b=4$ $b = \frac{5}{2}$ $a+b+c=3$ $\frac{1}{2} + \frac{5}{2} + c = 3$ $c = 0$ $T_n = \frac{1}{2}n^2 + \frac{5}{2}n$	$\checkmark 2a=1$ $\checkmark 3\left(\frac{1}{2}\right)+b=4$ $\checkmark \frac{1}{2} + \frac{5}{2} + c = 3$ <p style="text-align: right;">(3)</p>
<p>3.1.2</p>	$13\ 527 = \frac{1}{2}n^2 + \frac{5}{2}n$ $n^2 + 5n - 27\ 054 = 0$ $(n-162)(n+167) = 0$ $n = 162 \text{ or } n = -167$ $T_{161} = 13\ 363$ $\therefore T_{161} + 164 = 13\ 527$ <p>164 must be added.</p> <p>OR/OF</p> $T_n = 3 + \text{sum of 1}^{\text{st}} \text{ differences}$ $13527 = 3 + 4 + 5 + \dots + n$ $\frac{n-3+1}{2}[3+n] = 13527$ $n^2 + n - 27060 = 0$ $(n+165)(n-167) = 0$ $n = 164$	$\checkmark 13\ 527 = \frac{1}{2}n^2 + \frac{5}{2}n$ $\checkmark \text{standard form}$ $\checkmark \text{answers for } n$ $\checkmark 164$ <p style="text-align: right;">(4)</p> <p>OR/OF</p> $\checkmark 13527 = 3 + 4 + 5 + \dots + n$ $\checkmark n^2 + n - 27060 = 0$ $\checkmark \text{answers for } n$ $\checkmark 164$ <p style="text-align: right;">(4)</p>

3.2.1	$T_n = 8 + (n-1)(3)$ $T_n = 3n + 5$ $41 = 3n + 5$ $36 = 3n$ $n = 12$	✓ $T_n = 3n + 5$ ✓ $T_n = 41$ ✓ answer (3)
3.2.2a	$P_{41} = 12$	✓ answer (1)
3.2.2b	$P_8 = a + 7d = 1$ $P_{11} = a + 10d = 2$ $3d = 1$ $d = \frac{1}{3}$ $a + 7\left(\frac{1}{3}\right) = 1$ $a = -\frac{4}{3}$	✓ $a + 7d = 1$ ✓ $a + 10d = 2$ ✓ value of d ✓ value of a (4)
		[15]

QUESTION 2/VRAAG 2

<p>2.1.1</p>	<p>$7 + 12 + 17 + \dots$</p> <p>$T_n = a + (n-1)d$</p> <p>$T_{91} = 7 + (91-1)(5)$</p> <p>$T_{91} = 457$</p> <p>OR/OF</p> <p>$d = 5$</p> <p>$T_n = 5n + 2$</p> <p>$T_{91} = 5(91) + 2$</p> <p>$T_{91} = 457$</p>	<p>✓ $d = 5$</p> <p>✓ substitution into correct formula</p> <p>✓ answer (3)</p> <p>OR/OF</p> <p>✓ $d = 5$</p> <p>✓ substitution $n = 91$</p> <p>✓ answer (3)</p>
<p>2.1.2</p>	<p>$S_n = \frac{n}{2}[2a + (n-1)d]$</p> <p>$S_{91} = \frac{91}{2}[2 \times 7 + (91-1)(5)]$</p> <p>$S_9 = 21\ 112$</p> <p>OR/OF</p> <p>$S_n = \frac{n}{2}(a+l)$</p> <p>$S_{91} = \frac{91}{2}(7 + 457)$</p> <p>$S_{91} = 21\ 112$</p>	<p>✓ substitution into correct formula</p> <p>✓ answer (2)</p> <p>OR/OF</p> <p>✓ substitution into correct formula</p> <p>✓ answer (2)</p>
<p>2.1.3</p>	<p>$T_n = 7 + (n-1)(5)$</p> <p>$5n + 2 = 517$</p> <p>$5n = 515$</p> <p>$n = 103$</p>	<p>✓ substitution into correct formula</p> <p>✓ equate</p> <p>✓ answer (3)</p>
<p>2.2.1</p>	<p>$T_1 = 3; T_2 - T_1 = 9$ and $T_3 - T_2 = 21$</p> <p> $\begin{array}{cccccc} 3 & & 12 & & 33 & & 66 & & 111 \\ & \swarrow & & \swarrow & & \swarrow & & \swarrow & \\ & 9 & & 21 & & 33 & & 45 & \\ & & \swarrow & & \swarrow & & \swarrow & & \\ & & 12 & & 12 & & 12 & & \end{array}$ </p> <p>$\therefore T_5 = 3 + 9 + 21 + 33 + 45 = 111$</p> <p>OR/OF</p> <p>$2a = 12$</p> <p>$a = 6$</p> <p>$3(6) + b = 9$</p> <p>$b = -9$</p> <p>$6 - 9 + c = 3$</p> <p>$T_5 = 6(5)^2 - 9(5) + 6 = 111$</p>	<p>✓ constant second diff = 12</p> <p>✓ first differences : 33 and 45</p> <p>(2)</p> <p>OR/OF</p> <p>✓ constant second diff = 12</p> <p>✓ substitute 5 (2)</p>

QUESTION 2/VRAAG 2

2.1.1	$\frac{1}{5} + \frac{1}{15} + \frac{1}{45} + \dots$ $r = \frac{\frac{1}{15}}{\frac{1}{5}} = \frac{1}{3}$ $-1 < \frac{1}{3} < 1$ $\therefore \text{the series is convergent.}$	$\checkmark r = \frac{1}{3}$ $\checkmark \text{ answer (any indicator of convergence) (2)}$
2.1.2	$S_{\infty} = \frac{a}{1-r}$ $= \frac{\frac{1}{5}}{1 - \frac{1}{3}}$ $= \frac{3}{10}$	$\checkmark \text{ substitution}$ $\checkmark \text{ answer (2)}$
2.2.1	$4x; \frac{1}{81}$	$\checkmark 4x \quad \checkmark \frac{1}{81} \quad (2)$
2.2.2	$T_n = x + (n-1)x$ $= x + xn - x$ $= xn$	<div style="border: 1px solid black; padding: 5px; display: inline-block;">Answer only: Full Marks</div> $\checkmark \text{ substitution}$ $\checkmark \text{ answer (2)}$
2.2.3	$T_n = ar^{n-1}$ $T_{13} = \frac{1}{3} \left(\frac{1}{3} \right)^{13-1}$ $T_{13} = \left(\frac{1}{3} \right)^{13} \text{ or } \frac{1}{1\,594\,323} \text{ or } 6,27 \times 10^{-7} \text{ or } 3^{-13}$	$\checkmark n = 13$ $\checkmark r = \frac{1}{3}$ $\checkmark \text{ answer (3)}$
2.2.4	$\sum_{n=1}^{21} P_n = S_{11} + S_{10}$ $= \frac{11}{2} [2x + 10x] + \frac{\frac{1}{3} \left[1 - \left(\frac{1}{3} \right)^{10} \right]}{1 - \frac{1}{3}}$ $= 66x + 0,5$ $33,5 = 66x + 0,5$ $\therefore x = \frac{1}{2}$	$\checkmark S_{11} \quad \checkmark + S_{10}$ $\checkmark \text{ arithmetic sum}$ $\checkmark \text{ geometric sum}$ $\checkmark 66x + 0,5 \quad (\text{A})$ $\checkmark \text{ answer (6)}$
		[17]

QUESTION 3/VRAAG 3

<p>3.1</p>	$ \begin{array}{ccc} x & ; & x & ; & T_3 & ; & \dots \\ & \swarrow & & \searrow & & & \\ & 0 & & T_3 - x & & & \\ & & \swarrow & & \searrow & & \\ & & & & & & 10 \end{array} $ <p> $2a = 10 \quad 3a + b = 0$ $a = 5 \quad b = -15$ </p> <p> $T_3 - x - 0 = 10$ $\therefore T_3 = x + 10$ </p> <p> $2x + T_3 = 28$ $2x + x + 10 = 28$ $3x = 18$ $x = 6$ </p> <p> $a + b + c = 6$ $5 - 15 + c = 6$ $c = 16$ </p> <p> $\therefore T_n = 5n^2 - 15n + 16$ </p> <p>OR/OF</p> <p> $2a = 10$ $\therefore a = 5$ </p> <p> $T_1 = a + b + c \quad T_2 = 4a + 2b + c \quad T_3 = 9a + 3b + c$ $= 5 + b + c \quad = 20 + 2b + c \quad = 45 + 3b + c$ </p> <p> $5 + b + c = 20 + 2b + c$ $b = -15$ </p> <p> $T_1 = -10 + c \quad T_2 = -10 + c \quad T_3 = c$ </p> <p> $T_1 + T_2 + T_3 = -10 + c - 10 + c + c$ $28 = 3c - 20$ $c = 16$ </p>	<p> $\checkmark 2a = 10$ $\checkmark 3a + b = 0$ </p> <p> $\checkmark T_3 = x + 10$ </p> <p> $\checkmark 2x + T_3 = 28$ </p> <p> $\checkmark x = 6$ </p> <p> $\checkmark 5 - 15 + c = 6$ </p> <p style="text-align: right;">(6)</p> <p>OR/OF</p> <p> $\checkmark 2a = 10$ </p> <p> $\checkmark 5 + b + c = 20 + 2b + c$ </p> <p> $\checkmark T_1 = -10 + c$ $\checkmark T_2 = -10 + c$ </p> <p> $\checkmark 28 = 3c - 20$ $\checkmark c = 16$ </p> <p style="text-align: right;">(6)</p>
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NOVEMBER 2022

QUESTION 2/VRAAG 2

2.1.1	$a = 14$ $T_6 = 14r^5 = 448$ $r^5 = 32$ $\therefore r = 2$	<div style="border: 1px solid black; padding: 2px; display: inline-block; margin: 5px;">Answer only: full marks</div> $\checkmark T_6 = 14r^5 = 448$ $\checkmark r = 2$ (2)
2.1.2	$T_n = 14(2)^{n-1}$ $S_n = \frac{14(2^6 - 1)}{2 - 1}$ $S_6 = 882$ $114\ 674 - 882 = 113\ 792$ $113\ 792 = 896(2^n - 1)$ $128 = 2^n$ $n = 7$ OR/OF $S_n = \frac{a(r^n - 1)}{r - 1}$ $114\ 674 = \frac{14(2^n - 1)}{2 - 1}$ $8\ 191 = 2^n - 1$ $2^n = 8\ 192$ $n = \log_2 8\ 192$ $n = 13$ $\therefore 7$ more terms must be added to the first 6 terms.	\checkmark substitution into correct formula $\checkmark S_6 = 882$ $\checkmark 128 = 2^n$ $\checkmark 7$ (4) OR/OF \checkmark substitution into correct formula $\checkmark 2^n = 8\ 192$ $\checkmark n = 13$ $\checkmark 7$ (4)
2.1.3	$r = \frac{1}{2}$ OR $448r^5 = 14$ $\therefore r = \frac{1}{2}$ $S_\infty = \frac{a}{1 - r}$ $S_\infty = \frac{448}{1 - \frac{1}{2}}$ $S_\infty = 896$	$\checkmark r = \frac{1}{2}$ \checkmark substitution \checkmark answer (3)

<p>2.2</p> $\sum_{p=0}^k \left(\frac{1}{3}p + \frac{1}{6} \right) = 20 \frac{1}{6}$ $T_1 = \frac{1}{6} \quad T_2 = \frac{1}{3} + \frac{1}{6} = \frac{3}{6}$ $d = \frac{3}{6} - \frac{1}{6} = \frac{1}{3}$ $\frac{121}{6} = \frac{n}{2} \left[2 \left(\frac{1}{6} \right) + (n-1) \left(\frac{1}{3} \right) \right]$ $\frac{121}{3} = n \left[\frac{1}{3} + \frac{1}{3}n - \frac{1}{3} \right]$ $\frac{121}{3} = \frac{1}{3}n^2$ $121 = n^2$ $n = 11$ $\therefore k = 10$ <p>OR/OF</p> $\sum_{p=0}^k \left(\frac{1}{3}p + \frac{1}{6} \right) = 20 \frac{1}{6}$ $a = \frac{1}{6}$ $l = \frac{1}{3}k + \frac{1}{6}$ $n = k + 1$ $S_n = \frac{n}{2} [a + l]$ $\frac{121}{6} = \frac{k+1}{2} \left[\frac{1}{6} + \frac{1}{3}k + \frac{1}{6} \right]$ $\frac{121}{6} = \frac{k+1}{2} \left[\frac{1}{3}k + \frac{1}{3} \right]$ $\frac{121}{6} = \frac{k+1}{2} \left[\frac{k+1}{3} \right]$ $\frac{121}{6} = \frac{(k+1)^2}{6}$ $k+1 = \pm \sqrt{121}$ $k+1 = 11$ $k = 10$	<p>✓ $T_1 = \frac{1}{6}$</p> <p>✓ d</p> <p>✓ substitution</p> <p>✓ value of n</p> <p>✓ value of k (5)</p> <p>OR/OF</p> <p>✓ $a = \frac{1}{6}$</p> <p>✓ l</p> <p>✓ $n = k + 1$</p> <p>✓ $\frac{121}{6} = \frac{(k+1)^2}{6}$</p> <p>✓ value of k (5)</p>	<p>(5)</p> <p>(5)</p> <p>(5)</p> <p>(5)</p>
[14]		

QUESTION 3/VRAAG 3

3.1	$3a + b = 7$ $3 + b = 7$ $b = 4$ OR/OF $T_2 - T_1 = 7$ $4 + 2b + 9 - (1 + b + 9) = 7$ $b = 4$	$\checkmark 3a + b = 7$ $\checkmark 3 + b = 7$ (2) OR/OF $\checkmark T_2 - T_1 = 7$ \checkmark substitution (2)
3.2	$T_n = n^2 + 4n + 9$ $T_{60} = (60)^2 + 4(60) + 9$ $= 3849$ <div style="border: 1px solid black; display: inline-block; padding: 2px;">Answer only: full marks</div>	\checkmark substitution \checkmark answer (2)
3.3	14 ; 21 ; 30 ; 41 ; First difference: 7 ; 9 ; 11 ; ... Common 2 nd difference: 2 $T_p = 2p + 5$ <div style="border: 1px solid black; display: inline-block; padding: 2px;">Answer only: full marks</div> OR/OF First difference: 7 ; 9 ; 11 ; ... $T_n = a + (n-1)d$ $T_p = 7 + (p-1)(2)$ $T_p = 2p + 5$	\checkmark first difference $\checkmark 2$ $\checkmark 2p + 5$ (3) OR/OF \checkmark first difference $\checkmark 2$ $\checkmark 2p + 5$ (3)
3.4	$157 = 2p + 5$ $p = 76$ \therefore Between T_{76} and T_{77} OR/OF $T_{n+1} - T_n = 157$ $(n+1)^2 + 4(n+1) + 9 - (n^2 + 4n + 9) = 157$ $n^2 + 2n + 1 + 4n + 4 + 9 - n^2 - 4n - 9 = 157$ $2n = 152$ $n = 76$ \therefore Between T_{76} and T_{77}	$\checkmark 157 = 2p + 5$ $\checkmark p = 76$ $\checkmark T_{76}$ and T_{77} (3) OR/OF $\checkmark T_{n+1} - T_n = 157$ $\checkmark n = 76$ $\checkmark T_{76}$ and T_{77} (3)
		[10]

QUESTION/VRAAG 2

2.1.1	$a + 6d = 35$ $-1 + 6d = 35$ $6d = 36$ $d = 6$ OR/OF $\frac{35 - (-1)}{7 - 1} = 6$	<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 0 auto;">ANSWER ONLY: FULL MARKS</div> ✓ substitution ✓ answer (2) OR/OF ✓ substitution ✓ answer (2)
2.1.2	$T_n = a + (n - 1)d$ $473 = -1 + (n - 1)(6)$ $79 = n - 1$ $\therefore n = 80$	<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 0 auto;">ANSWER ONLY: FULL MARKS</div> ✓ substitution into the correct formula ✓ equating to 473 ✓ answer (3)
2.1.3	$S_n = \frac{n}{2}[2a + (n - 1)d]$ $S_{40} = \frac{40}{2}[2(-1) + (40 - 1)(6)]$ $\therefore S_{40} = 4\,640$ OR/OF $T_{40} = 6(40) - 7$ $= 233$ $S_n = \frac{n}{2}(a + l)$ $= \frac{40}{2}(-1 + 233)$ $= 4\,640$	✓ substitution ✓ answer (2) OR/OF ✓ substitution ✓ answer (2)
2.2.1	$75 \quad \quad \quad 53 \quad \quad \quad 35 \quad \quad \quad 21$ $\quad \quad \quad \backslash \quad / \quad \quad \backslash \quad / \quad \quad \backslash \quad /$ $\quad \quad \quad -22 \quad \quad \quad -18 \quad \quad \quad -14$ $\quad \quad \quad / \quad \backslash \quad \quad / \quad \backslash \quad \quad / \quad \backslash$ $\quad \quad \quad 4 \quad \quad \quad 4$ $T_5 = 11$	✓ answer (A) (1)
2.2.2	$T_n = an^2 + bn + c$ $2a = 4$ $a = 2$ $3a + b = -22$ $6 + b = -22$ $b = -28$ $a + b + c = 75$ $2 - 28 + c = 75$ $c = 101$ $\therefore T_n = 2n^2 - 28n + 101$	✓ $T_n = an^2 + bn + c$ ✓ $a = 2$ ✓ $b = -28$ ✓ $c = 101$ (4)

2.2.3

Minimum value of T_n

$$n = -\frac{b}{2a} = -\frac{(-28)}{2(2)}$$

$$n = 7$$

$$\text{Minimum value of } T_n = 2(7)^2 - 28(7) + 101 = 3$$

Each term in the new pattern is $-\frac{1}{5}$ the value of the terms in the old pattern.

$$\text{Maximum value of new pattern} = -\frac{3}{5}$$

OR/OF

$$T'_n = 4n - 28$$

$$4n - 28 = 0$$

$$4n = 28$$

$$n = 7$$

$$\text{Minimum value of } T_n = 2(7)^2 - 28(7) + 101 = 3$$

Each term in the new pattern is $-\frac{1}{5}$ the value of the terms in the old pattern.

$$\text{Maximum value of new pattern} = -\frac{3}{5}$$

OR/OF

$$T_n = -\frac{2}{5}n^2 + \frac{28}{5}n - \frac{101}{5}$$

$$n = -\frac{b}{2a} = \frac{-\frac{28}{5}}{2\left(\frac{-2}{5}\right)}$$

$$= 7$$

$$T_7 = -\frac{3}{5}$$

$$\checkmark n = 7$$

$$\checkmark \text{ min value} = 3$$

$\checkmark -\frac{1}{5}$ value of term of old pattern

$$\checkmark \text{ max value} = -\frac{3}{5} \quad (4)$$

OR/OF

$$\checkmark n = 7$$

$$\checkmark \text{ min value} = 3$$

$\checkmark -\frac{1}{5}$ value of term of old pattern

$$\checkmark \text{ max value} = -\frac{3}{5} \quad (4)$$

OR/OF

$$\checkmark \checkmark T_n \div (-5)$$

$$\checkmark n = 7$$

$$\checkmark \text{ max value} = -\frac{3}{5} \quad (4)$$

QUESTION/VRAAG 3

3.1.1	$T_n = ar^{n-1}$ $T_{10} = 1024 \left(\frac{1}{4}\right)^{10-1}$ $\therefore T_{10} = \frac{1}{256}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p style="text-align: center;">ANSWER ONLY: FULL MARKS</p> </div>	<p>✓ substitution into the correct formula</p> <p>✓ answer</p> <p style="text-align: right;">(2)</p>
3.1.2	$\sum_{p=0}^8 256(4^{1-p}) = 1024 + 256 + 64 + \dots$ $S_n = \frac{a[1-r^n]}{1-r}$ $S_9 = \frac{1024 \left[1 - \left(\frac{1}{4}\right)^9\right]}{1 - \frac{1}{4}}$ $S_9 = \frac{87381}{64}$ $= 1365,33$ <p>OR/OF</p> $\sum_{p=0}^8 256(4^{1-p})$ $= 1024 + 256 + 64 + 16 + 4 + 1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64}$ $S_9 = \frac{87381}{64}$ $= 1365,33$	<p>✓ 1024</p> <p>✓ $n = 9$</p> <p>✓ substitution into the correct formula</p> <p>✓ answer</p> <p style="text-align: right;">(4)</p> <p>OR/OF</p> <p>✓ 1024</p> <p>✓ rest of expansion</p> <p>✓ $n = 9$ terms</p> <p>✓ answer</p> <p style="text-align: right;">(4)</p>
3.2	$-t^2 - 6t - 9; \frac{t^3 + 9t^2 + 27t + 27}{2}$ $-(t^2 + 6t + 9); \frac{1}{2}(t+3)(t^2 + 6t + 9)$ $-(t+3)^2; \frac{1}{2}(t+3)^3$ $r = \frac{-(t+3)}{2}$ $-1 < \frac{-t-3}{2} < 1$ $-2 < -t-3 < 2$ $1 < -t < 5$ $-5 < t < -1$	$\frac{t^3 + 9t^2 + 27t + 27}{-t^2 - 6t - 9}$ <p>✓ $r = \frac{2}{-t^2 - 6t - 9}$</p> <p>✓ $-(t^2 + 6t + 9)$</p> <p>✓ $\frac{1}{2}(t+3)(t^2 + 6t + 9)$</p> <p>✓ $-1 < \frac{-t-3}{2} < 1$</p> <p>✓ answer</p> <p style="text-align: right;">(5)</p>
		[11]